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Logica Universalis



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Logical Squares for Classical Logic Sentences

Urszula Wybraniec-Skardowska

Abstract. In this paper, with reference to relationships of the traditional square of opposition, we establish all the relations of the square of opposition between complex sentences built from the 16 binary and four unary propositional connectives of the classical propositional calculus (CPC). We illustrate them by means of many squares of opposition and, corresponding to them—octagons, hexagons or other geometrical objects.

Mathematics Subject Classification. Primary 03B05, 03B65; Secondary 03B80.

Keywords. Square of opposition, classical propositional connectives, truth-value table, tautology of classical logic, octagon of opposition, hexagon of opposition, octahedron of opposition.

1. Introduction: Basic Definitions

For any sentences $\alpha, \beta, \varphi, \psi$ of CPC we assume the following definitions:

α, β are *contrary* iff α/β is a tautology of CPC, where the stroke/ is the Sheffer's connective;

φ, ψ are *subcontrary* iff $\varphi \vee \psi$ is a tautology of CPC, where \vee is the disjunction connective;

α entails φ iff $\alpha \rightarrow \varphi$ is a tautology of CPC, where \rightarrow is the implication connective;

α and ψ are *contradictory* iff $(\alpha \wedge \psi) \vee (\sim \alpha \wedge \sim \psi)$ is a counter-tautology of CPC, i.e. α and ψ never agree in truth-values.

We will illustrate the above relationships between sentences $\alpha, \beta, \varphi, \psi$ in a square of opposition graphically in a non-standard way¹ by means of Fig. 1, where the dotted lines indicate contradictory sentences and the downward arrows the implication.

¹ First logicians who tried to organize or 'structure' the connectives of CPC in a systematic way were [3, 5, 7, 11]. A description of the paper by [5] is given by Beziau [2]: <http://cahiers.kingston.ac.uk/synopses/syn10.7.html>.

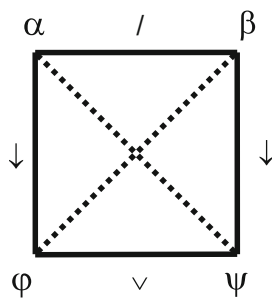


FIGURE 1. Scheme of square of opposition

2. Connectives: both ... and ...; ... unless ...; not ... because ...; neither ... nor ...²

Among the 16 binary classical sentence-forming connectives there are only four for which sentences that are built by means of them are true only in one case (see Table 1). They are the following:

both ... and ..., ... unless ..., not ... because ..., neither ... nor ... (binegation \swarrow).

We define them by means of variables p, q and the classical connectives: the conjunction \wedge (or implication \rightarrow) and negation \sim as follows:

both p and $q =_{df} p \wedge q$;

p unless $q =_{df} \sim(p \rightarrow q) \equiv p \wedge \sim q$;

not p because q (or not p though q) $=_{df} \sim(q \rightarrow p) \equiv \sim p \wedge q$;

neither p nor q ($p \swarrow q$) $=_{df} \sim p \wedge \sim q$.

The third connective is also called the *dual implication*: $d(p \rightarrow q) =_{df} \sim(q \rightarrow p)$ (see [16, 17]) and the last one is known as *binegation* \swarrow .

The truth-value table for these connectives is the following:

TABLE 1. The truth-value table for conjunctive sentences

p	q	$p \wedge q$	$p \wedge \sim q$	$\sim p \wedge q$	$\sim p \wedge \sim q$
1	1	1	0	0	0
1	0	0	1	0	0
0	1	0	0	1	0
0	0	0	0	0	1

The above conjunctive sentences (conjunctions) are pairwise *contrary*, i.e. the Sheffer's disjunction of two sentences of each of the six pairs of the above

² These connectives of natural language fulfill in sentences not only a logical, descriptive (communicative) role but also an expressive one (expressing psychical states of a speaker). Defining these connectives by means of well-known logical connectives of CPC, we omit of course their expressive functions in composed propositions.

conjunctions is true (is a tautology), so the sentences can never both be true, but can both be false.

To each of the 6 pairs of *contrary* conjunctions from the following:

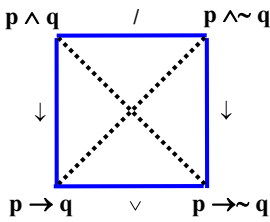
- (i) $p \wedge q$, $p \wedge \sim q$, $\sim p \wedge q$, $\sim p \wedge \sim q$

there exists a pair of *contradictory* implications from the following pairwise *subcontrary* implications:

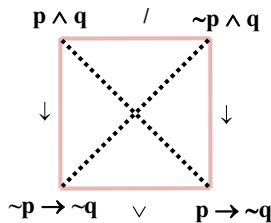
- (ii) $p \rightarrow q$, $p \rightarrow \sim q$, $\sim p \rightarrow q$, $\sim p \rightarrow \sim q$

so the sentences that can both be true, but cannot both be false, i.e. the disjunction of which is true. The implications (ii), of course, are true in three cases depending on the truth value of sentences p and q .

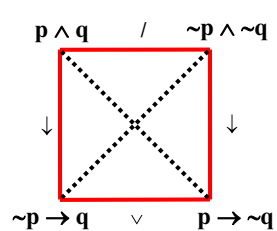
Each pair of (ii), together with the suitable pair of contrary conjunctions of (i), creates one of the 6 squares of opposition for complex sentences of classical logic. The squares are given below (see Squares 1–6).



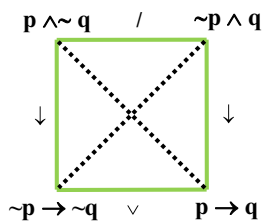
Square 1.



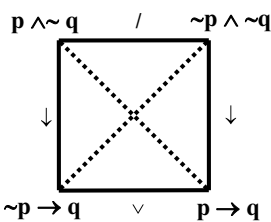
Square 2.



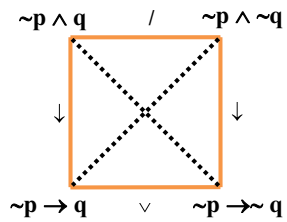
Square 3.



Square 4.



Square 5.



Square 6.

To each of the 6 pairs of *contrary* conjunctions from the following:

- (i) $p \wedge q$, $p \wedge \sim q$, $\sim p \wedge q$, $\sim p \wedge \sim q$

there exists a pair of *contradictory* Sheffer's disjunctions (denial alternatives) from the following pairwise *subcontrary* Sheffer's disjunctions, i.e. the disjunction of which is true:

- (iii) p/q , $p/\sim q$, $\sim p/q$, $\sim p/\sim q$.

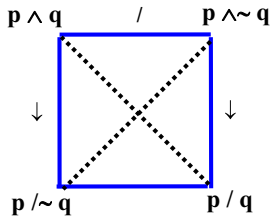
Each pair of (iii) together with the suitable pair of conjunctions of (i), creates one of the six squares of opposition for complex sentences of classical logic (see Squares 1'–6').³

To each of the six pairs of *contrary* conjunctions from (i) there exists also a pair of *contradictory* disjunctions from the following pairwise *subcontrary* disjunctions, i.e. the disjunction of which is true:

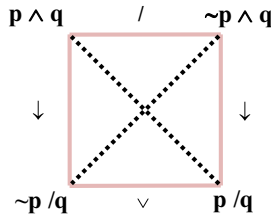
$$(iv) \quad p \vee q, p \vee \sim q, \sim p \vee q, \sim p \vee \sim q.$$

Each pair of (iv), juxtaposed with the suitable pair of conjunctions from (i), creates also one of the six the squares of opposition for complex classical sentences (see Squares 1''–6'').⁴

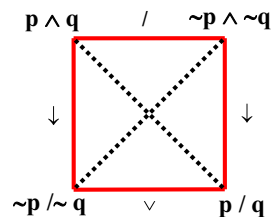
Squares 1'–6' and 1''–6'' are given below:



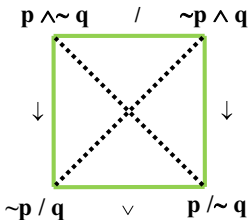
Square 1'.



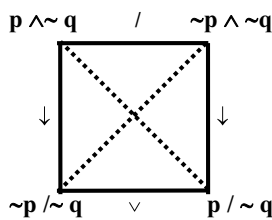
Square 2'.



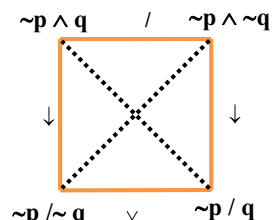
Square 3'.



Square 4'.



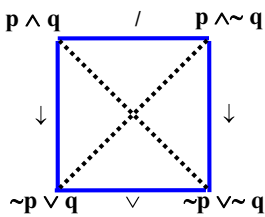
Square 5'.



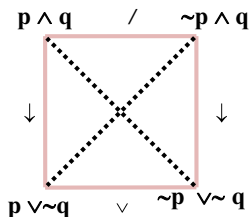
Square 6'.

³ Squares 1'–6' are formed with the suitable Squares 1–6 by replacing in them implications by equivalent Sheffer's disjunctions.

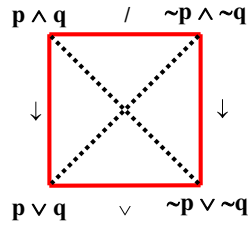
⁴ Squares 1''–6'', are formed with the suitable Squares 1–6 (resp. Squares 1'–6') by replacing in them implications (resp. Sheffer's disjunctions) by equivalent disjunctions.



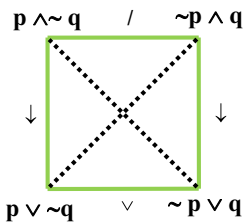
Square 1''



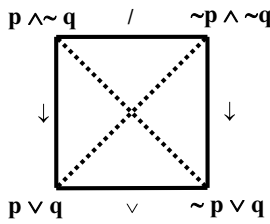
Square 2''.



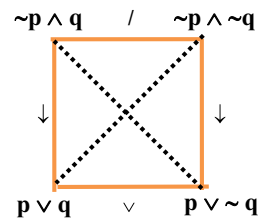
Square 3''.



Square 4''.

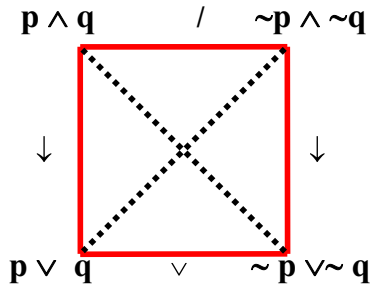


Square 5''.



Square 6''.

As it turns out, at least one of them, Square 3'':



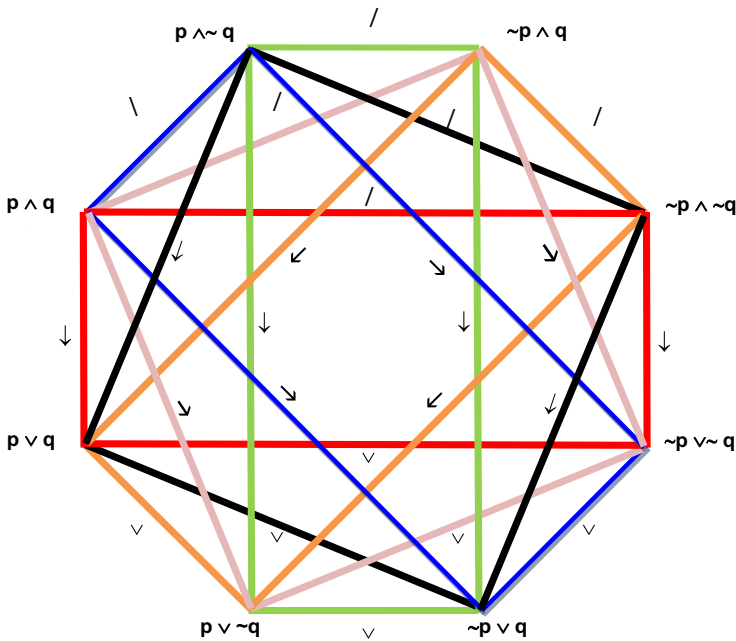
was known much earlier (see [4, 7, 11–13, 19, 21],⁵ [8].

So, if we have, for example, $p \approx$ John is a scientist, $q \approx$ John is a priest, then their conjunction: John is a scientist *and* John is a priest **is contrary to** their bination: *neither* John is a scientist *nor* John is a priest; their disjunction: John is a scientist *or* John is a priest **is subcontrary to** the disjunction of their negations; their conjunction **is contradictory to** the disjunction of their negations and their bination **is contradictory to** their disjunction;

⁵ A fragment of the book of Żarnecka Biały (pp. 65–66) devoted to the theory of opposition and to Petrus Hispanus (Pope John XXII, in 13th century) suggests that the square was known to him. However, as Wojciech Suchoń informed me, in Petrus Hispanus's *Summulae logicales* there is no reference to the square of opposition for complex sentences, although it contains some material that allows for such digressions (cf. my abstract in [18]).

from their conjunction **follows** their disjunction and from their bination **follows** disjunction of their negations.

We may illustrate each of the six squares of opposition: Squares 1–6 (resp. Squares 1'–6' or Squares 1''–6''), and properly corresponding to them six rectangles of opposition, in one of the octagons of opposition: Octagon 1 (resp. Octagon 2 or Octagon 3). For Squares 1''–6'' we have Octagon 3 in which, for clarity, we omitted the dotted, diagonal lines indicating contradictory sentences.



Octagon 3.

This octagon is another representation of the so called logical cube (well known in literature) which constitutes the central part of bigger 3D representations which unite all 14/16 formulas of CPC. It occurs in [8, 9, 13, 14].

For Squares 1–6 and 1'–6' we obtain Octagon 1 and Octagon 2 in which disjunctions of (iv) are either replaced by equivalent implications from (ii) or by equivalent Sheffer's disjunctions from (iii), respectively.

Among the 16 connectives of the set F16 of the all binary connectives of classical logic we may find six which form true sentences in two cases and one which forms a true sentence in four cases. We will consider them in the next sections.

3. Connectives: ...even if...; even if...,...; ...if and only if...; either...or...

3.1. The six binary classical sentence-forming connectives that form true sentences in two cases are defined as follows:

p even if q ($p <| q$) $=_{df}$ $q \vee \sim q \rightarrow p \equiv p$; $\sim p$ even if $\sim q$ ($\sim p <| \sim q$)
 $=_{df}$ $q \vee \sim q \rightarrow \sim p \equiv \sim p$;
 even if p, q ($p |> q$) $=_{df}$ $p \vee \sim p \rightarrow q \equiv q$; even if $\sim p, \sim q$ ($\sim p |> \sim q$)
 $=_{df}$ $p \vee \sim p \rightarrow \sim q \equiv \sim q$;
 p if and only if q ($p \equiv q$) $=_{df}$ $(p \wedge q) \vee (\sim p \wedge \sim q)$; either p or q ($p \perp q$)
 $=_{df}$ $\sim (p \equiv q)$.

We see that sentences in the same line on the right side and on the left side are contradictory.

The connective \perp ($\vee\vee, \vee$) is well known as the strong or exclusive disjunction connective. The sentence: even if p, q ($p |> q$) can be read: even if p then q , and the sentence: even if $\sim p, \sim q$ ($\sim p |> \sim q$) can be read: even if not p then not q .

The truth-value table for the connectives $<|, |>, \equiv$ and \perp is the following (see Table 2)⁶:

TABLE 2. The truth-value table for connectives: $<|, |>, \equiv, \perp$

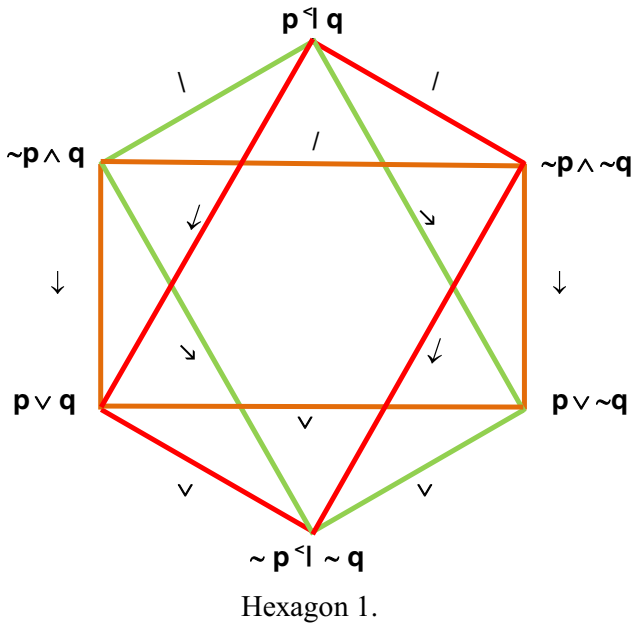
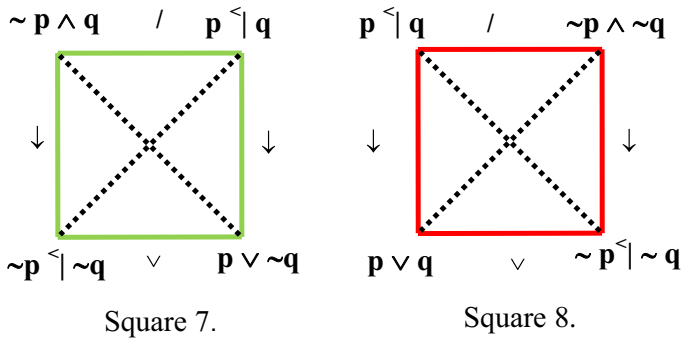
p	q	$p < q$	$\sim p < \sim q$	$p > q$	$\sim p > \sim q$	$p \equiv q$	$p \perp q$
1	1	1	0	1	0	1	0
1	0	1	0	0	1	0	1
0	1	0	1	1	0	0	1
0	0	0	1	0	1	1	0

In every two successive columns with composed propositions we have two contradictory sentences. For each pair of contradictory sentences in Table 2 we can build four squares of opposition and two hexagons of opposition corresponding to them.

The idea of constructing hexagons built from the squares of oppositions (rectangles of opposition corresponding to them) differs from the main idea of Blanché's hexagon (1966) presented clearly by Béziau [1] and based on putting together two triangles of opposition: the triangle of contrariety and the triangle of subcontrariety.

⁶ The sentence $\sim p <| \sim q$ can be replaced by the sentence $\sim p <| q$ and the sentence $\sim p |> \sim q$ —by the sentence $p |> \sim q$.

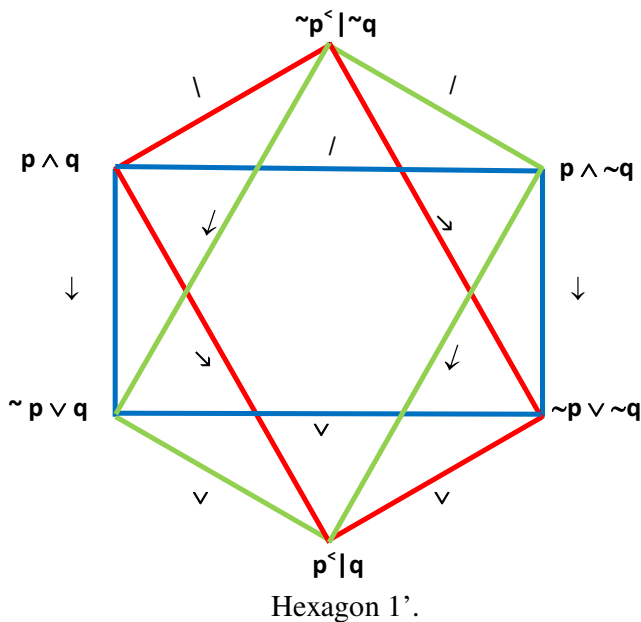
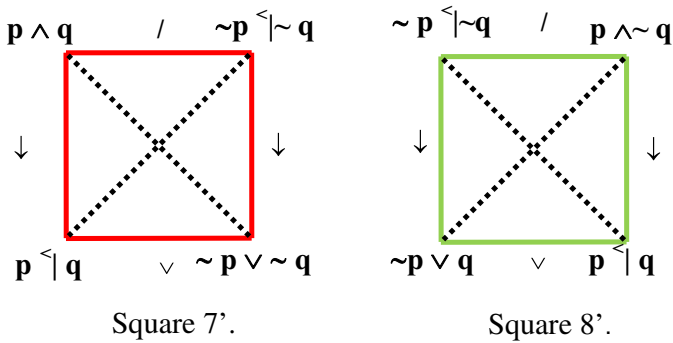
3.2. For $p <| q$ and $\sim p <| \sim q$ ($\sim p <| q$) we have Squares 7 and 8 and Hexagon 1 (in which Square 6'' is also illustrated) and Squares 7' and 8' and Hexagon 1' (in which Square 1'' is illustrated).



We obtain Hexagon 1 by *composition* of Squares 7 and 8. The crucial difference between this Hexagon and the classical one from [4] is that two triangles of contrariety and subcontrariety do not overlap, and that all entailment relations are systematically pointed downwards, all contraries relations are shown in its upper triangle and all subcontraries relations on its bottom triangle. Hexagon 1 is a simple graphical variant of the standard Blanche hexagon which is obtained by flipping the vertical diagonal of contradiction upside down.⁷

⁷ I owe this remark to the reviewer of my paper.

Hexagon 1' is obtained here by *composition* of the above mentioned Squares 7' and 8':



Let us return to the first square of the series of squares. Let us consider an example of using Square 7 with contradictory sentences $p < q$ and $\sim p < \sim q$.

Let p be a sentence: *He goes for a walk*, and q a sentence: *It rains*. Then

- the sentence $p < q$: *He goes for a walk even if it rains-*

is contrary to

the sentence $\sim p \wedge q$: *He does not go for a walk because (and) it rains;*

- the sentence $p < q$: *He goes for a walk even if it rains*

is contradictory to

the sentence $\sim p < \sim q$: *He does not go for a walk even if it does not rain;*

- the sentence $\sim p \wedge q$: *He does not go for a walk because (and) it rains*
is contradictory to

the sentence $p \vee \sim q$: *He goes for a walk or it does not rain;*

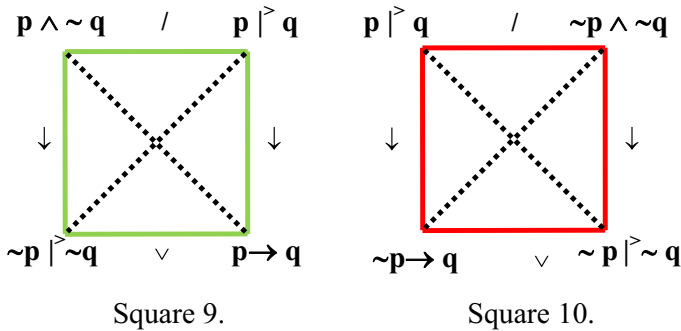
- the sentence $p <| q$: *He goes for a walk even if it rains*
implies the sentence $p \vee \sim q$: *He goes for a walk or it does not rain;*

- the sentence $\sim p \wedge q$: *He does not go for a walk because (and) it rains*
implies the sentence $\sim p <| \sim q$: *He does not go for a walk even if it does not rain;*

- the sentence $p \vee \sim q$: *He goes for a walk or it does not rain* and
the sentence $\sim p <| \sim q$: *He does not go for a walk even if it does not rain*
are subcontrary.

3.3. For contradictory sentences $p > q$ and $\sim p > \sim q$ ($p > \sim q$) we have Squares 9 and 10 and Hexagon 2, and also Squares 9' and 10' and Hexagon 2'.

Below we have Squares 9 and 10:



Let us consider an example of using Square 9 with contradictory sentences $p > q$ and $\sim p > \sim q$. Let us recall that the sentence: p unless $q =_{\text{df}} \sim (p \rightarrow q) \equiv p \wedge \sim q$. And let p be a sentence: *The reviewer rejected this paper*, and q a sentence: *It will be published*.

Then

- the sentence $p > q$: *Even if the reviewer rejected this paper, it will be published*
is contrary to
the sentence $p \wedge \sim q$ (p unless q , $\sim (p \rightarrow q)$): *It is not truth that if the reviewer rejected this paper it will be published;*

- the sentence $p \mid\!> q$: Even if the reviewer rejected this paper, it will be published,

is contradictory to

the sentence $\sim p \mid\!> \sim q$: Even if the reviewer did not reject this paper, it will not be published;

- the sentence $p \wedge \sim q$: The reviewer rejected this paper unless it will be published

is contradictory to

the sentence $p \rightarrow q$: If the reviewer rejected this paper then it will be published;

- the sentence $p \mid\!> q$: Even if the reviewer rejected this paper, it will be published

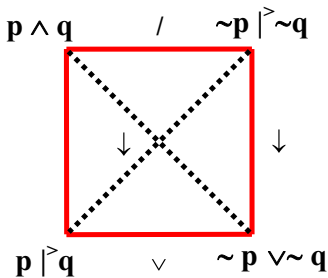
implies the sentence $p \rightarrow q$: If the reviewer rejected this paper then it will be published;

- the sentence $p \wedge \sim q$: The reviewer rejected this paper unless it will be published

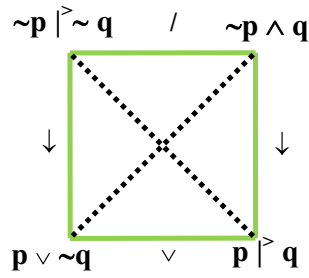
implies the sentence $\sim p \mid\!> \sim q$: Even if the reviewer did not reject this paper, it will not be published;

- the sentence $p \rightarrow q$: If the reviewer rejected this paper then it will be published, and the sentence $\sim p \mid\!> \sim q$: Even if the reviewer did not reject this paper, it will not be published, **are subcontrary**.

Below we present Squares 9' and 10':

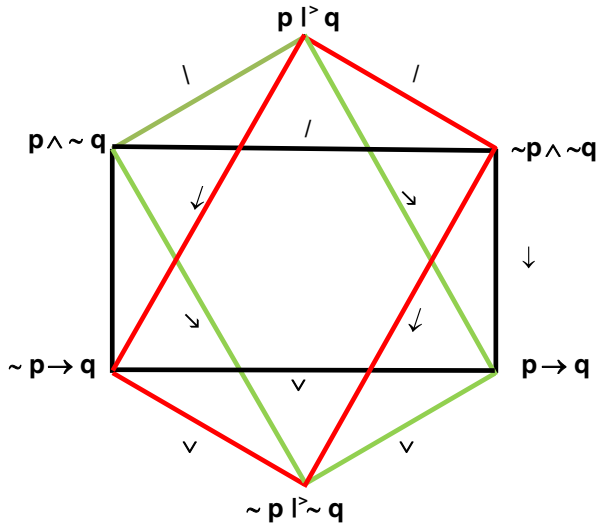


Square 9'.

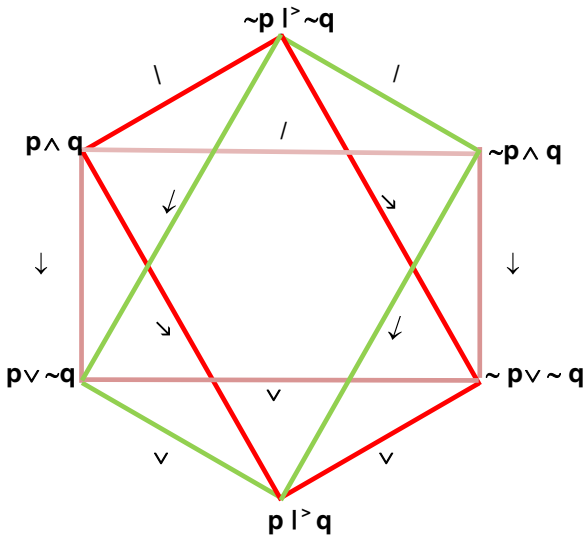


Square 10'.

By *composition* of Squares 9 and 10 (resp. Squares 9' and 10') we obtain Hexagon 2 (resp. Hexagon 2'):



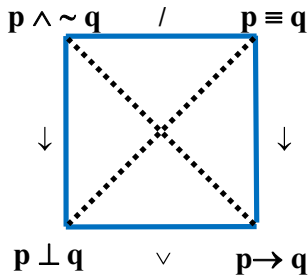
Hexagon 2.



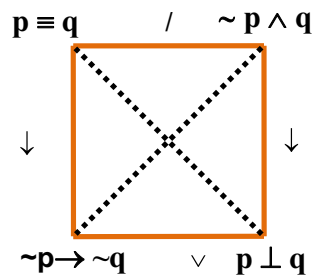
Hexagon 2'.

In Hexagon 2 and Square 5 is also illustrated, while in Hexagon 2' and Square 2'' is presented.

3.4. For contradictory sentences $p \equiv q$ and $p \perp q$ we have Squares 11 and 12 and Hexagon 3 (in which Square 4 is also illustrated) and also Squares 11' and 12' and Hexagon 3' (in which Square 3 is also presented).

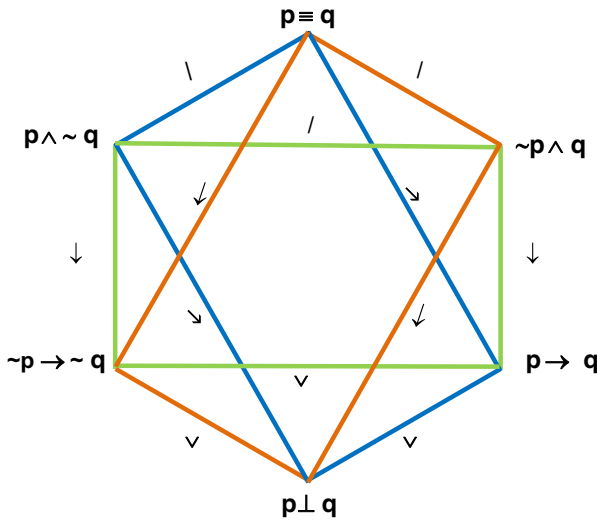


Square 11.



Square 12.

The composition of Squares 11 and 12 is Hexagon 3:



Hexagon 3.

Let us consider an example using the relationships in Square 12. Let us assume that the sentence $p \approx$ *This paper will be published*, $q \approx$ *This paper has good reviews*. Let us also recall that the sentence: not p because $q =_{df} \sim (q \rightarrow p) \equiv \sim p \wedge q$. So

- the sentence $p \equiv q$: *This paper will be published if and only if this paper has good reviews **is contrary to***

the sentence $\sim p \wedge q$: *This paper will not be published though it has good reviews;*

- the sentence $p \equiv q$: *This paper will be published if and only if this paper has good reviews **is contradictory to***

the sentence $p \perp q$: *Either this paper will be published or it has good reviews;*

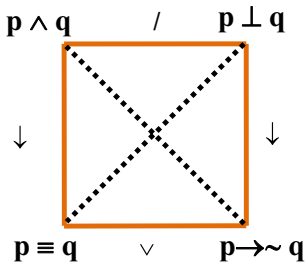
- the sentence $p \equiv q$: *This paper will be published if and only if this paper has good reviews **implies** the sentence $q \rightarrow p$: If this paper has good reviews then it will be published;*
- the sentence $\sim p \wedge q$: *This paper will not be published though it has good reviews*

is contradictory to

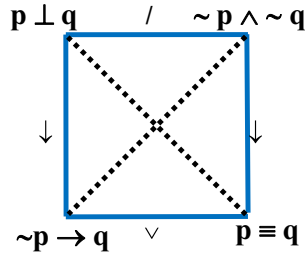
the sentence $q \rightarrow p$: If this paper has good reviews then it will be published;

- the sentence $\sim p \wedge q$: *This paper will not be published though it has good reviews*
- **implies** the sentence $p \perp q$: Either this paper will be published or it has good reviews.

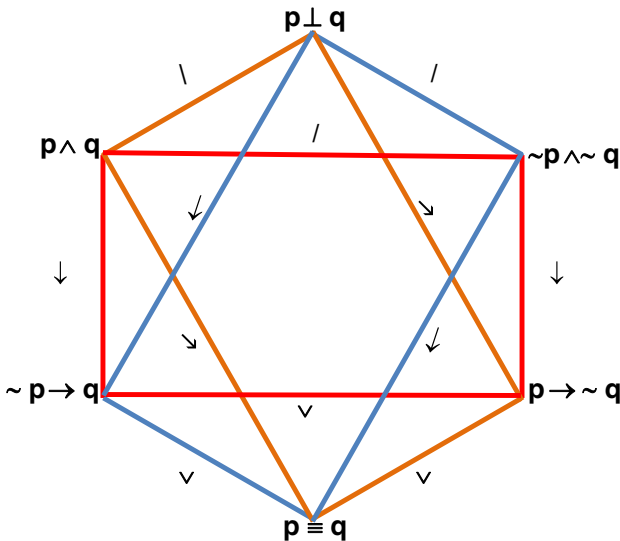
Squares 11' and 12' and Hexagon 3' are given below.



Square 11'.



Square 12'.



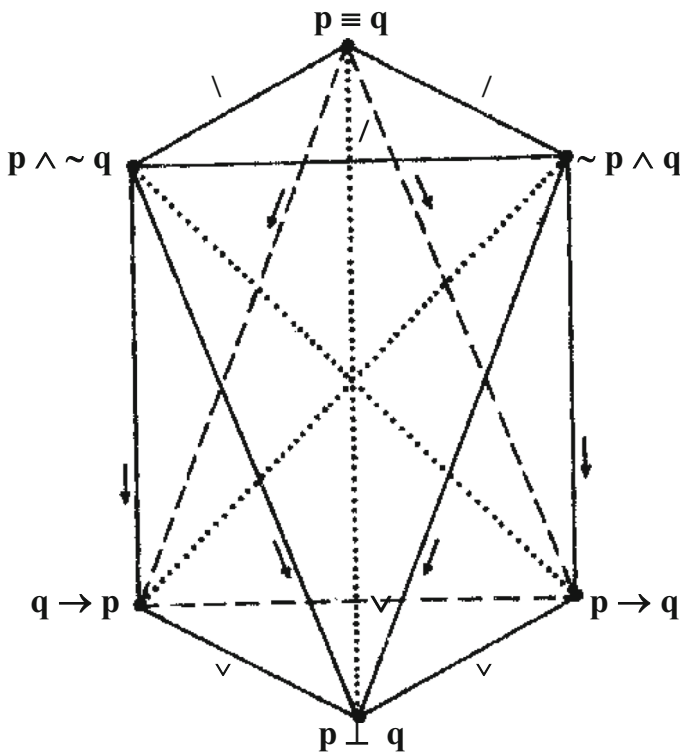
Hexagon 3'.

3.5. The relationships illustrated by means of Hexagons 1–3 can be shown by means of three octahedrons of opposition, respectively. The relationships

illustrated by means of Hexagons 1'–3' can be also shown by means of three other octahedrons of opposition, respectively.

In the book by [20] the relationships illustrated in Hexagon 3 are illustrated below by means of the so-called *Ośmiościan logiczny* (*Logical Octahedron*). Żarnecka-Biały used in the octahedron α, β for p, q ; dotted lines—for indicating contradictory sentences; the sign $\&$ —for the conjunction connective \wedge , and the symbol \vee with the dot on top—for the strong disjunctive connective \perp . It should, however, be observed that instead of $\alpha \rightarrow \beta$ there should be $\beta \rightarrow \alpha$, and reversely.

The figure depicted as Octahedron 3 is the Żarnecka-Biały's original *Logical Octahedron* picture given in our presented above changed symbolism.



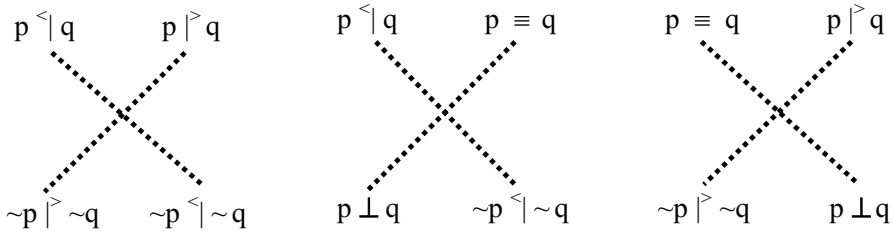
Octahedron 3.

In Octahedron 3 we have eight triangular faces: (1) $\triangle (q \rightarrow p, p \perp q, p \rightarrow q)$, (2) $\triangle (p \perp q, p \rightarrow q, \sim p \wedge q)$, (3) $\triangle (p \wedge \sim q, q \rightarrow p, p \perp q)$, (4) $\triangle (p \wedge \sim q, p \perp q, \sim p \wedge q)$, (5) $\triangle (p \wedge \sim q, p \equiv q, \sim p \wedge q)$, (6) $\triangle (q \rightarrow p, p \wedge \sim q, p \equiv q)$, (7) $\triangle (q \rightarrow p, p \equiv q, p \rightarrow q)$, (8) $\triangle (p \equiv q, \sim p \wedge q, p \rightarrow q)$.

We can easily get other octahedrons of opposition: Octahedron 1 and Octahedron 2, corresponding to Hexagon 1 and Hexagon 2, respectively, and Octahedrons 1', 2', 3' corresponding to Hexagons 1', 2' and 3', respectively.

3.6. “Degenerate” Squares

On the basis of composed formulas in Table 2 it is possible to generate three more squares, which are “degenerate” in that they only preserve the diagonals of contradiction, but lose all the other relations (contrariety, subcontrariety and subalternation):



This type of squares⁸ is also called “unconnectedness square” (see [15]) and is called by Béziau the “X of oppositions” (2012).

4. Binary Connectives: *verum* and *falsum*

So far we have considered:

Four binary connectives which form true classical sentences in one case (see Table 1; Sect. 2),

Six binary connectives which form true sentences in two cases (see Table 2; Sect. 3).

Four implicational connectives which form sentences given in (ii)

[resp. equivalent Sheffer’s disjunctive connectives which form sentences given in (iii) or equivalent disjunctive connectives forming propositions given in (iv)] that form true sentences in three cases.

All these 14 connectives of the set F16 of all the binary connectives of CPC were taken into consideration while forming some sentences which help to establish appropriate relationships in the squares of opposition given above.

So it remains to consider what relationships are between all of the 14 distinguished connectives of the set F16 and the connectives:

It is always truth independently of p and q [symbolically: $T(p, q)$ or $p \top q$]

and

It is always falsity independently of p and q [symbolically: $F(p, q)$ or $p \bot q$]

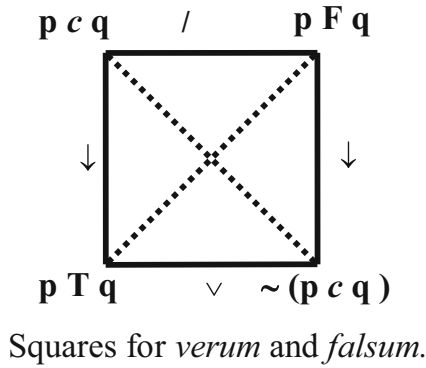
called here *binary verum* and *binary falsum*, respectively. They are defined in Table 3.

⁸ The squares were indicated by the reviewer of this paper.

TABLE 3. The truth-value table for connectives *verum* T and *falsum* F

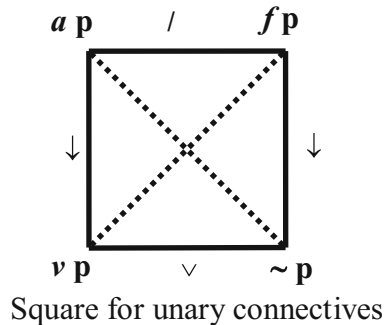
p	q	p T q	p F q
1	1	1	0
1	0	1	0
0	1	1	0
0	0	1	0

For these contradictory connectives *verum* T and *falsum* F, together with each connective $c \neq T$ and $c \neq F$ of the set F16, we may build 14 squares of opposition in the following form:



5. Square for Unary Connectives

The last squares correspond to the known square of opposition for unary connectives: *assertion* a , *falsum* f , *verum* v and *negation* \sim , which has the following form:



6. Conclusion

In Sects. 2–5 we were able to consider all the squares of opposition relationships for all sentences built from connectives of classical logic (CPC): 16 binary and 4 unary.⁹

In Sect. 2 we showed that on the basis of four composed sentences in Table 1 we can built six basic squares of opposition with numbers 1–6 (so called “balanced” squares in literature). In Sect. 3 we showed that on the basis of Table 2 we can built 12 squares of opposition, so call “unbalanced” squares, with numbers 7–12 and 7’–12’ (and 6 hexagons with numbers 1–3 and 1’–3’). Moreover, it is also possible to generate three more “degenerate” squares (see Sect. 3.6).

Generally, we can say that on the basis of Tables 1 and 2, CPC contains 21 squares of opposition.

In Sect. 4 we considered additionally 14 squares of opposition for the binary connectives: *verum* and *falsum*. In Sect. 5 we also present one square of opposition for unary connectives of CPC.

As we could see, replacing categorical sentences in the traditional square of opposition with the suitable formulas of the classical propositional calculus (CPC) is a fully justified generalization of the idea of the logical square of opposition. This idea was known earlier ever since Blanché and Sauriol in the literature but based on other methods presented squares of oppositions for CPC.

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⁹ A different (“tetrahedral”) approach to the description of the relations among the 16 binary connectives is presented in the paper of Dubois and Prade [6, Section 2.3]. Another (tetrahexahedral) solution is presented by Moretti [10, Figs. 15, 19]. The standard tetrahexahedron is presented earlier in [13] as well as [8].

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Urszula Wybraniec-Skardowska
Department of Philosophy
Cardinal Stefan Wyszyński University in Warsaw
01-938 Warsaw
Poland
e-mail: skardowska@gmail.com

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